

Inertial spin Hall effect in noncommutative space

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In the present paper the study of inertial spin current(that appears in an accelerated frame of reference) is extended to Non-Commutative (NC) space. The θ -dependence, (θ being the NC parameter), of the inertial spin current is derived explicitly. We have provided yet another way of experimentally measuring θ . Our bound on θ matches with previous results. In Hamiltonian framework, the Dirac Hamiltonian in an accelerating frame is computed in the low energy regime by exploiting the Foldy-Wouthuysen scheme. The NC θ -effect appears from the replacement of normal products and commutators by Moyal $*$ -products and $*$ -commutators. In particular, the commutator between the external magnetic vector potential and the potential induced by acceleration becomes non-trivial. Expressions for θ -corrected inertial spin current and conductivity are derived. The θ bound is obtained from the out of plane spin polarization, which is experimentally observable.

PACS numbers: 03.65.-w, 71.70.Ej, 02.40.Gh

I. INTRODUCTION:

Systems living in Non-Commutative (NC) spacetime have created a lot of interest in recent years (for extensive reviews see e.g. [1]). Although the study of noncommutative geometry has a long history [2], it was revived in a different perspective after the seminal work of Seiberg and Witten [3] that showed its relevance in certain low energy limits of String Theory. This work also paved the way for a systematics framework where quantum field theories in classical (or commutative) spacetime can be extended to NC spacetimes via the Seiberg-Witten map [3]. This map expresses a quantum field in NC spacetime in terms of a quantum field in commutative spacetime in the form of a power series in the NC parameter. In a relativistic framework, the spacetime noncommutativity would be represented by $[x_\mu, x_\nu] = i\theta_{\mu\nu}$. For arbitrary $\theta_{\mu\nu}$, higher (than quadratic) order time derivatives will be generated through the Seiberg-Witten map, leading to a possible violation of unitarity. To avoid this type of pathology it is convenient to restrict the noncommutativity to the spatial sector only (i.e. $\theta_{0\mu} = 0$), with the following phase space algebra,

$$[x_i, x_j] = i\theta_{ij}, [x_i, p_j] = i\hbar\delta_{ij}, [p_i, p_j] = 0. \quad (1)$$

θ_{ij} an anti-symmetric real tensor, is the NC parameter. It should be mentioned that for (spacetime) constant $\theta_{\mu\nu}$ Lorentz invariance is violated. An interesting alternative framework has been provided in [4], (with recent applications in [5]), where $\theta_{\mu\nu}$ is elevated to a Lorentz tensor so that Lorentz invariance of the NC algebra is maintained although the possibility of loss of unitarity from higher time derivatives through Seiberg-Witten map will remain. We have used the simpler NC model of constant θ_{ij} (1) primarily because we finally consider low energy non-relativistic scenario. Operationally effect of the Seiberg-Witten map on a quantum field theory is encoded simply by replacing products of quantum field at the same spacetime point by Moyal (or $*$) product as defined below,

$$f(\vec{x}) * g(\vec{x}) = \exp\left[-\frac{i\theta_{ij}}{2}\partial_{x_i}\partial_{y_j}\right]f(\vec{x})g(\vec{y})|_{x=y}. \quad (2)$$

In the above $f(\vec{x})$ and $g(\vec{x})$ are two generic fields in commutative space. Clearly the NC effects generated from the $*$ -product appear as higher derivative correction terms and in the limit $\theta_{ij} \rightarrow 0$ the NC theory smoothly reduces to the original theory in commutative space. Indeed the algebra (1) is consistent with (2) where commutators are replaced by $*$ -commutators,

$$[A, B]_* = A * B - B * A \quad (3)$$

In low-energy limit, by considering the one-particle sector of field theory on noncommutative space one arrives at NC extension of quantum mechanics. To match with present experimental bounds, NC effects are expected to be

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very small. Even then it is interesting to study various effects of NC space on conventional systems. (For different examples we cite some of the works in [6].)

In this paper, we wish to discuss the effect of NC space on inertial spin current and spin polarization. The study of spin current is of recent interest from different perspectives [7–13]. It were Dyakonov and Perel [14], who first predicted the spin Hall effect (SHE) and the effect was theoretically developed in [15]. This effect is observed experimentally in semiconductors and metals [16]. It is a form of anomalous Hall effect induced by spin. Here a beam of particles separates in to up and down spin projections in the presence of perpendicular electric field in analogy to Hall effect where charges are separated in a beam passing through a perpendicular magnetic field. SHE, occurs due to the spin orbit coupling(SOC) of electron, which is the relativistic coupling of electron spin with orbital motion, become an active area of research[7–13]. However, the studies on the inertial effect of electrons has a long history [17–20] but the contribution of the spin-orbit coupling in accelerating frames has not much been addressed in the literature [21, 22]. In the literature we find that there has been an elegant attempt to extend the theory of spin current in the inertial frame [23]. A theory has been proposed describing the direct coupling of the mechanical rotation and spin current and predicting the spin current generation arising from rotational motion.[23, 24]. Recently, Spin Hall effect in NC space has been studied in [25–28].

Let us put the present work in its proper perspective. Apart from extending previous results of inertial effects of spin current and conductivity in NC space, (that is indeed of interest), the main motivation was to provide a bound for θ -the NC parameter involving an experimentally measurable parameter. Indeed, we have succeeded in relating θ to the out of plane spin polarization(s_z), which is a measurable quantity.

In order to address the above-mentioned issues, in the present work, it is quite interesting to explore the conditions of the spin-dependent inertial forces and induced spin currents that appear in non-inertial accelerating frames [24] placed in the NC space.

The paper is organized as follows. Section II deals with a linearly accelerating frame in NC space. The formalism adopted by us is explained in detail. Here we have used the Foldy Wouthuysen transformation [30] to get the non relativistic limit of Dirac equation. We have dealt with the idea [24] of interpreting the effect of linear acceleration on an electron as induced effective electric field in this section. The spin-orbit coupling resulting from this induced electric field along with electric field due to the spin-orbit coupling generated from the external electromagnetic field and noncommutativity produces the spin current in our system. Here we follow the physically intuitive approach of Chudnovsky [31], based on an extension of the Drude model and derive the spin Hall current and conductivity in a NC framework. In section III we explain in detail how the noncommutativity affects the Rashba [32] like coupling parameter and the spin polarization. Lastly in section IV we propose a new bound for the NC parameter θ from the expression of out of plane spin polarization. The paper ends with Conclusions in section V.

II. INERTIAL SPIN ORBIT COUPLING IN PRESENCE OF NON COMMUTATIVITY

Our general framework is the following: we first construct the Dirac equation in a non-inertial frame, following the work of Hehl and Ni [20]. Subsequently we introduce the NC effects. The essential idea in [20] is to introduce a system of orthonormal tetrad carried by the accelerating observer. This in turn induces a non-trivial metric and subsequently one rewrites the Dirac equation in the observer's local frame where normal derivatives are replaced by covariant derivatives derived from the induced metric. A series of Foldy-Wouthuysen Transformations (FWT) [30] yield a non-relativistic approximation of the Dirac Hamiltonian. Finally NC effects are incorporated by extending the commutators appearing in FWT to $*$ -commutators defined in (3).

The Dirac Hamiltonian in an arbitrary non-inertial frame with linear acceleration and rotation is given by [20]

$$H = \beta mc^2 + c \left(\vec{\alpha} \cdot \left(\vec{p} - \frac{e\vec{A}}{c} \right) \right) + \frac{1}{2c} \left[(\vec{a} \cdot \vec{r}) \left(\left(\vec{p} - \frac{e\vec{A}}{c} \right) \cdot \vec{\alpha} \right) + \left(\left(\vec{p} - \frac{e\vec{A}}{c} \right) \cdot \vec{\alpha} \right) (\vec{a} \cdot \vec{r}) \right] \\ + \beta m (\vec{a} \cdot \vec{r}) + eV(\vec{r}) - \vec{\Omega} \cdot (\vec{L} + \vec{S}) \quad (4)$$

where \vec{a} and $\vec{\Omega}$ are respectively the linear acceleration and rotation frequency of the observer with respect to an inertial frame. \vec{L} and \vec{S} are respectively the angular momentum ($\vec{L} = \vec{r} \times \vec{p}$) and spin of the Dirac particle. \vec{A} denotes the vector potential. The Dirac matrices β , α and the spin operator Σ for 4-spinor are respectively given by

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \quad \Sigma = \frac{\hbar}{2} \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} \quad (5)$$

In this section, we drop the rotation term $\vec{\Omega} \cdot (\vec{L} + \vec{S})$ to study only the effects of linear acceleration. For further calculations one has to apply FWT [20, 30] on the Hamiltonian in (4).

Let us outline the Foldy-Wouthuysen transformation (FWT) for NC space in the present case. The Dirac wave function is a four component spinor with the up and down spin electron and hole components. Generically the energy gap between the electron and hole is much larger than the energy scales associated with condensed matter systems. Hence it is natural to take the non relativistic limit of Dirac equation. One can achieve this by block diagonalization method of the Dirac Hamiltonian exploiting FWT [30]. For $\vec{\Omega} = 0$ in (4) the Hamiltonian can be divided into block diagonal and off diagonal parts denoted by ϵ and O respectively. Thus the Hamiltonian can be written as

$$\begin{aligned} H &= \beta mc^2 + O + \epsilon, \\ O &= c \left(\vec{\alpha} \cdot \left(\vec{p} - \frac{e\vec{A}}{c} \right) \right) + \frac{1}{2c} \left[(\vec{a} \cdot \vec{r}) \left(\left(\vec{p} - \frac{e\vec{A}}{c} \right) \cdot \vec{\alpha} \right) + \left(\left(\vec{p} - \frac{e\vec{A}}{c} \right) \cdot \vec{\alpha} \right) (\vec{a} \cdot \vec{r}) \right], \\ \epsilon &= \beta m(\vec{a} \cdot \vec{r}) + eV(\vec{r}). \end{aligned} \quad (6)$$

where m is the mass of the Dirac particle and $\beta = \gamma_0$, $\alpha_i = \gamma_0 \gamma_i$ are the Dirac matrices. Applying FWT on H yields,

$$H_{FW} = \beta \left(mc^2 + \frac{O^2}{2mc^2} \right) + \epsilon - \frac{1}{8m^2 c^4} [O, [O, \epsilon]]. \quad (7)$$

Now comes the novel part of our work. As we have explained in the Introduction, NC space effects can be incorporated simply by replacing the (space dependent) products and brackets in the Hamiltonian (7) by $*$ -products and $*$ -brackets (in particular $O^2 \rightarrow O * O$, $[O, \epsilon] \rightarrow [O, \epsilon]_*$). We restrict ourselves to $O(\theta)$ results and find,

$$\begin{aligned} O^2 &= c^2 \frac{\left(\left(\vec{p} - \frac{e\vec{A}}{c} \right) * \left(\vec{p} - \frac{e\vec{A}}{c} \right) \right)}{2m} - ce\hbar \vec{\Sigma} \cdot \vec{B} + ie^2 \vec{\Sigma} \cdot (\vec{A} \times_* \vec{A}) \\ [O, \epsilon]_* &= -ice\hbar \vec{\alpha} \cdot \vec{\nabla} V(\vec{r}) - e^2 \vec{\alpha} \cdot [\vec{A}, V(\vec{r})]_* - \beta m i c \hbar (\vec{\alpha} \cdot \vec{a}) - \beta m e \vec{\alpha} \cdot [\vec{A}, \vec{a} \cdot \vec{r}]_* \\ [O, [O, \epsilon]_*]_* &= ec^2 \hbar^2 (\vec{\nabla} \cdot \vec{E}) + ie c^2 \hbar^2 \vec{\Sigma} \cdot (\vec{\nabla} \times \vec{E}) + 2ec^2 \hbar \vec{\Sigma} \cdot (\vec{E} \times \vec{p}) - 2ie^2 c \vec{\Sigma} \cdot ([\vec{A}, V(\vec{r})]_* \times \vec{p}) \\ &\quad - \beta m c^2 \hbar^2 (\vec{\nabla} \cdot \vec{a}) - i\beta m c^2 \hbar^2 \vec{\Sigma} \cdot (\vec{\nabla} \times \vec{a}) - 2\beta m c^2 \hbar \vec{\Sigma} \cdot (\vec{a} \times \vec{p}) + 2i\beta c m e \Sigma \cdot ([\vec{A}, \vec{a} \cdot \vec{r}]_* \times \vec{p}) \end{aligned} \quad (8)$$

Terms like $[\vec{A}, [\vec{A}, V(\vec{r})]_*]_*$, $\vec{\nabla} \cdot [\vec{A}, V(\vec{r})]_*$, $\vec{\nabla} \cdot [\vec{A}, \vec{a} \cdot \vec{r}]_*$ etc are $O(\theta^2)$ and hence dropped. The redshift effect of kinetic energy ([20]) is also neglected in the above calculations. Adding all these terms, the FW transformed Hamiltonian on the NC space takes the form,

$$\begin{aligned} H_{FW*} &= \beta \left(mc^2 + \frac{(p - \frac{e\vec{A}}{c}) * (p - \frac{e\vec{A}}{c})}{2m} \right) + eV(\vec{r}) + \beta m(\vec{a} \cdot \vec{r}) \\ &\quad - \frac{e\hbar}{2mc} \vec{\Sigma} \cdot \vec{B} - \frac{e\hbar^2}{8m^2 c^2} (\vec{\nabla} \cdot \vec{E}) - \frac{ie\hbar^2}{8m^2 c^2} \vec{\Sigma} \cdot (\vec{\nabla} \times \vec{E}) - \frac{e\hbar}{4m^2 c^2} \vec{\Sigma} \cdot (\vec{E} \times \vec{p}) \\ &\quad + \frac{\beta \hbar^2}{8mc^2} (\vec{\nabla} \cdot \vec{a}) + \frac{i\beta \hbar^2}{8mc^2} \vec{\Sigma} \cdot (\vec{\nabla} \times \vec{a}) + \frac{\beta \hbar}{4mc^2} \vec{\Sigma} \cdot (\vec{a} \times \vec{p}) - \frac{\beta e \hbar}{2mc} \vec{\Sigma} \cdot \vec{B}_\theta + \frac{e\hbar}{4m^2 c^2} \vec{\Sigma} \cdot (\vec{E}_\theta \times \vec{p}) - \frac{i\beta e}{4mc^3} \vec{\Sigma} \cdot ([\vec{A}, \vec{a} \cdot \vec{r}]_* \times \vec{p}) \end{aligned} \quad (10)$$

Let us simplify the Hamiltonian a little bit. As we are dealing with the constant acceleration we can drop the terms $(\vec{\nabla} \cdot \vec{a})$ and $\vec{\Sigma} \cdot (\vec{\nabla} \times \vec{a})$. Consideration of constant electric field can help us leaving the terms with $(\vec{\nabla} \times \vec{E})$ and $(\vec{\nabla} \cdot \vec{E})$. Finally, we land up with the Hamiltonian as

$$\begin{aligned} H_{FW*} &= \left(mc^2 + \frac{\left((p - \frac{e\vec{A}}{c}) * (p - \frac{e\vec{A}}{c}) \right)}{2m} \right) + eV(\vec{r}) - \frac{e\hbar}{2mc} \vec{\sigma} \cdot \vec{B} + m(\vec{a} \cdot \vec{r}) \\ &\quad - \frac{e\hbar}{4m^2 c^2} \vec{\sigma} \cdot (\vec{E} \times \vec{p}) + \frac{\hbar}{4mc^2} \vec{\sigma} \cdot (\vec{a} \times \vec{p}) \\ &\quad + \frac{e\hbar}{4m^2 c^2} \vec{\sigma} \cdot (\vec{E}_\theta \times \vec{p}) - \frac{e\hbar}{4m^2 c^2} \vec{\sigma} \cdot (\vec{E}_{\vec{a}, \theta} \times \vec{p}) - \frac{e\hbar}{2mc} \vec{\sigma} \cdot \vec{B}_\theta. \end{aligned} \quad (11)$$

where $\vec{E}_\theta = -\frac{ie}{c\hbar}[\vec{A}, V]_*$, and $\vec{B}_\theta = -\frac{ie}{c\hbar}(\vec{A} \times_* \vec{A})$ and $\vec{E}_{\vec{a},\theta} = -\frac{ie}{c\hbar}[\vec{A}, V_a(\vec{r})]_*$. Here we consider a potential $V_a = -\frac{m}{e}\vec{a} \cdot \vec{r}$. We make a point here that the inertial effect of the linear acceleration on electron can be interpreted as an induced *effective electric field* $\vec{E}_{\vec{a}}$ such that

$$\vec{E}_{\vec{a}} = \frac{m}{e}\vec{a}, \quad (12)$$

(where the induced electric field $\vec{E}_{\vec{a}}$ is the gradient of some potential V_a). Introduction of this effective electric field $\vec{E}_{\vec{a}}$ generates an inertial spin-orbit term and its NC correction, apart from the spin-orbit term arising due to the external electric field (fifth term in the right hand side of (11)). The above FW Hamiltonian on the NC space gives the dynamics of an electron (or hole with proper sign of e) in the positive energy part of the full energy spectrum. Here $\vec{\sigma}$ is the Pauli spin matrix. To study the dynamics of the charged particles, the vector potential \vec{A} plays an important role, which can be understood via the Aharonov-Bohm effect [33]. The choice of \vec{A} , the vector potential, will impose the condition on \vec{B} and $\vec{B}_\theta = \vec{A} \times_* \vec{A}$. The choice put in our analysis will be stated later.

$$\begin{aligned} H_{FW*} = & \left(mc^2 + \frac{\left((p - \frac{e\vec{A}}{c}) * (p - \frac{e\vec{A}}{c}) \right)}{2m} \right) + eV(\vec{r}) - \frac{e\hbar}{2mc}\vec{\sigma} \cdot \vec{B} - eV_a(\vec{r}) \\ & - \frac{e\hbar}{4m^2c^2}\vec{\sigma} \cdot (\vec{E} \times \vec{p}) + \frac{e\hbar}{4m^2c^2}\vec{\sigma} \cdot (\vec{E}_{\vec{a}} \times \vec{p}) \\ & + \frac{e\hbar}{4m^2c^2}\vec{\sigma} \cdot (\vec{E}_\theta \times \vec{p}) - \frac{e\hbar}{4m^2c^2}\vec{\sigma} \cdot (\vec{E}_{\vec{a},\theta} \times \vec{p}) - \frac{e\hbar}{2mc}\vec{\sigma} \cdot \vec{B}_\theta. \end{aligned} \quad (13)$$

We are now in a position to explain the underlying physics of the individual terms of the right hand side of the Hamiltonian (13). The first two terms describe the relativistic mass increase, whereas the third term is the electrostatic energy and the fourth term is a magnetic dipole energy which induces Zeeman effect. The fifth term arises due to linear acceleration in the system. The terms $\vec{\sigma} \cdot (\vec{E} \times \vec{p})$, $\vec{\sigma} \cdot (\vec{E}_{\vec{a}} \times \vec{p})$ and $\vec{\sigma} \cdot (\vec{E}_\theta \times \vec{p})$, $\vec{\sigma} \cdot (\vec{E}_{\vec{a},\theta} \times \vec{p})$ are respectively the spin-orbit interaction terms and its correction due to noncommutativity.

To derive the equations of motion of the electron we follow the physically intuitive approach of Chudnovsky [31], based on an extension of the Drude model and its NC extension [26]. Collecting the dynamical terms and the terms due to spin orbit coupling, the final Hamiltonian for the positive energy solution of spin $\frac{1}{2}$ electron can now be read as

$$\begin{aligned} H_{FW*} = & \frac{p^2}{2m} + eV(\vec{r}) - eV_a(\vec{r}) - \frac{e\hbar}{4m^2c^2}\vec{\sigma} \cdot (\vec{E} \times \vec{p}) + \\ & \frac{e\hbar}{4m^2c^2}\vec{\sigma} \cdot (\vec{E}_{\vec{a}} \times \vec{p}) + \frac{e\hbar}{4m^2c^2}\vec{\sigma} \cdot (\vec{E}_\theta \times \vec{p}) - \frac{e\hbar}{4m^2c^2}\vec{\sigma} \cdot (\vec{E}_{\vec{a},\theta} \times \vec{p}) \end{aligned} \quad (14)$$

As we can see, in (14) we have neglected the rest energy term. Let us remind here that the space dependent potential $V(\vec{r})$ is the sum of the external electric potential $V_0(\vec{r})$ and the lattice electric potential $V_l(\vec{r})$. On the non commutative space the Heisenberg's equations of motion can be obtained in the standard method as [31]

$$\dot{\vec{r}} = \frac{1}{i\hbar}[\vec{r}, H_{FW*}], \quad (15)$$

$$\dot{\vec{p}} = \frac{1}{i\hbar}[\vec{p}, H_{FW*}]. \quad (16)$$

Consequently, we have

$$\begin{aligned} \dot{\vec{r}} = & \frac{\vec{p}}{m} + \frac{e\hbar}{4m^2c^2}(\vec{\sigma} \times \vec{\nabla} V(\vec{r})) - \frac{ie^2}{4m^2c^3}(\vec{\sigma} \times [\vec{A}, V(\vec{r})]_*) - \frac{e\hbar}{4m^2c^2}(\vec{\sigma} \times \vec{\nabla} V_a) + \frac{ie^2}{4m^2c^3}(\vec{\sigma} \times [\vec{A}, V_a(\vec{r})]_*) \\ \dot{\vec{p}} = & -e\vec{\nabla} V(\vec{r}) + e\vec{\nabla} V_a(\vec{r}) - \frac{e\hbar}{4m^2c^2}\vec{\nabla}((\vec{\sigma} \times \vec{\nabla} V(\vec{r})).\vec{p}) \\ & + \frac{ie^2}{4m^2c^3}\vec{\nabla}((\vec{\sigma} \times [\vec{A}, V(\vec{r})]_*).\vec{p}) + \frac{e\hbar}{4m^2c^2}\vec{\nabla}((\vec{\sigma} \times \vec{\nabla} V_a(\vec{r})).\vec{p}) - \frac{ie^2}{4m^2c^3}\vec{\nabla}((\vec{\sigma} \times [\vec{A}, V_a(\vec{r})]_*).\vec{p}) \end{aligned} \quad (17)$$

From (17) we can write

$$\vec{p} = m\vec{r} - \frac{e\hbar}{4mc^2}(\vec{\sigma} \times \vec{\nabla}V(\vec{r})) + \frac{ie^2}{4mc^3}(\vec{\sigma} \times [\vec{A}, V(r)]_*) + \frac{e\hbar}{4mc^2}(\vec{\sigma} \times \vec{\nabla}V_{\vec{a}}(\vec{r})) - \frac{ie^2}{4mc^3}(\vec{\sigma} \times [\vec{A}, V_{\vec{a}}(\vec{r})]_*) \quad (18)$$

The time derivative of (18) gives

$$\dot{\vec{p}} = m\dot{\vec{r}} - \frac{e\hbar}{4mc^2}(\dot{\vec{r}} \cdot \vec{\nabla})(\vec{\sigma} \times \vec{\nabla}V(\vec{r})) + \frac{ie^2}{4mc^3}(\dot{\vec{r}} \cdot \vec{\nabla})(\vec{\sigma} \times [\vec{A}, V(r)]_*) + \frac{e\hbar}{4mc^2}(\dot{\vec{r}} \cdot \vec{\nabla})(\vec{\sigma} \times \vec{\nabla}V_{\vec{a}}(\vec{r})) - \frac{ie^2}{4mc^3}(\dot{\vec{r}} \cdot \vec{\nabla})(\vec{\sigma} \times [\vec{A}, V_{\vec{a}}(\vec{r})]_*) \quad (19)$$

Finally, the equation of motion has the form

$$m\ddot{\vec{r}} = -e\vec{\nabla}V(\vec{r}) + e\vec{\nabla}V_{\vec{a}}(\vec{r}) - \frac{e\hbar}{4mc^2}\dot{\vec{r}} \times \vec{\nabla} \times (\vec{\sigma} \times \vec{\nabla}V(\vec{r})) + \frac{ie^2}{4mc^3}\dot{\vec{r}} \times \vec{\nabla} \times (\vec{\sigma} \times [\vec{A}, V(\vec{r})]_*) \\ + \frac{e\hbar}{4mc^2}\dot{\vec{r}} \times \vec{\nabla} \times (\vec{\sigma} \times \vec{\nabla}V_{\vec{a}}(\vec{r})) - \frac{ie^2}{4mc^3}\dot{\vec{r}} \times \vec{\nabla} \times (\vec{\sigma} \times [\vec{A}, V_{\vec{a}}(\vec{r})]_*) \quad (20)$$

or,

$$m\ddot{\vec{r}} = -e\vec{\nabla}(V(\vec{r}) - V_{\vec{a}}(\vec{r})) - \dot{\vec{r}} \times \vec{\nabla} \times \left(\frac{e\hbar}{4mc^2}(\vec{\sigma} \times \vec{\nabla}V(\vec{r})) - \frac{ie^2}{4mc^3}(\vec{\sigma} \times [\vec{A}, V(\vec{r})]_*) - \frac{e\hbar}{4mc^2}(\vec{\sigma} \times \vec{\nabla}V_{\vec{a}}(\vec{r})) + \frac{ie^2}{4mc^3}(\vec{\sigma} \times [\vec{A}, V_{\vec{a}}(\vec{r})]_*) \right) \quad (21)$$

It is worth mentioning here that this spin dependent effective Lorentz force noted in eqn.(21), is responsible for the transport of the electrons in the system on the NC space, and hence responsible for the spin current. As expected, when we put $\vec{a} = 0$ in eqn. (20), i.e the system is not accelerated, but put in the NC framework, our force equation is similar to that obtained in [26],

From the expression of $\dot{\vec{r}}$ in (17) we can write the linear velocity in linearly accelerating frame on non commutative space as

$$\dot{\vec{r}} = \frac{\vec{p}}{m} + \vec{v}_{\vec{\sigma}, \vec{a}, \theta} \quad (22)$$

where

$$\vec{v}_{\vec{\sigma}, \vec{a}, \theta} = \frac{e\hbar}{4m^2c^2}\vec{\sigma} \times \left[\vec{\nabla}V(\vec{r}) - \vec{\nabla}V_{\vec{a}}(\vec{r}) - \frac{ie}{c\hbar}[\vec{A}, V(\vec{r})]_* + \frac{ie}{c\hbar}[\vec{A}, V_{\vec{a}}(\vec{r})]_* \right] \\ = -\frac{e\hbar}{4m^2c^2}\vec{\sigma} \times \vec{\mathcal{E}}_{\vec{a}, \theta} \quad (23)$$

is the effective spin dependent velocity in the NC space with

$$\vec{\mathcal{E}}_{\vec{a}, \theta} = -\left[\vec{\nabla}V(\vec{r}) - \vec{\nabla}V_{\vec{a}}(\vec{r}) - \frac{ie}{c\hbar}[\vec{A}, V(\vec{r})]_* + \frac{ie}{c\hbar}[\vec{A}, V_{\vec{a}}(\vec{r})]_* \right] \quad (24)$$

being the *total effective electric field* present in the system. One should note here that the velocity term in (23) is dependent of the potential $\vec{\nabla}V(\vec{r})$ and $\vec{\nabla}V_{\vec{a}}(\vec{r})$, and also on their NC corrections. Thus the inertial effect on linear acceleration along with the non-commutativity produces the anomalous velocity term which in turn may yield the spin Hall effect for the particular choice of the vector potential \vec{A} .

Our chosen gauge $\vec{A} = (-x_2, x_1, 0)$ gives, $[\vec{A}, V(\vec{r})]_* = i\theta\vec{\nabla}V(\vec{r})$, $[\vec{A}, V_{\vec{a}}(\vec{r})]_* = i\theta\vec{\nabla}V_{\vec{a}}(\vec{r})$. Hence, we yield

$$\vec{\mathcal{E}}_{\vec{a}, \theta} = -\left[\vec{\nabla}V(\vec{r}) - \vec{\nabla}V_{\vec{a}} + \frac{\theta e}{c\hbar}\vec{\nabla}V(\vec{r}) - \frac{\theta e}{c\hbar}\vec{\nabla}V_{\vec{a}}(\vec{r}) \right]. \quad (25)$$

The polarized spin current due to the total effective electric field $\vec{\mathcal{E}}_{\vec{a}, \theta}$ is thus given by

$$\vec{j}_s = e n \text{Tr} \sigma_i \vec{v}_{\vec{\sigma}, \vec{a}, \theta}. \quad (26)$$

Thus the i^{th} component of the spin current on the NC space is given by

$$|j_s^i| = \frac{n e^2 \hbar}{2m^2 c^2} (\vec{S} \times \vec{\mathcal{E}}_{\vec{a}, \theta})^i \quad (27)$$

where \vec{S} is the spin vector. From this expression of current one can find out the spin current in different directions. This result can be compared with the result given in [24]. It can be easily checked from this expression that when $\theta = 0$, *i.e.* NC correction is absent, our result is in agreement with the results given in [24]. On the other hand, it can also be verified that when the system is not accelerated, but put in a NC framework our analysis is consistent with that of [26] with a different choice of vector potential.

The next job is to evaluate the explicit expression for the spin Hall conductivity in an accelerated system. With a careful observation of the force equation one can put further analysis on different components of the total current produced in an accelerating system moving in a NC space by adopting the averaging methodology followed in [31]. With our choice of gauge, one can write the force equation (21) as

$$m\ddot{\vec{r}} = \vec{F}_0 + \vec{F}_{\vec{\sigma}} \quad (28)$$

$$= \vec{F}_0 + F_1(\vec{\sigma}) + F_2(\vec{\sigma}, \theta) \quad (29)$$

where \vec{F}_0 is the spin independent part of the force and $\vec{F}_{\vec{\sigma}}$ is the spin dependent part of the total spin force, which actually corresponds to two parts, (i) spin dependent but θ independent part $\vec{F}_1(\vec{\sigma})$ and (ii) $\vec{F}_2(\vec{\sigma}, \theta)$ which is the noncommutative correction to the spin dependent part. The explicit forms of the above mentioned terms are

$$\vec{F}_0 = -e\vec{\nabla}V(\vec{r}) + e\vec{\nabla}V_a(r) = -e\vec{\nabla}V_{tot}, \quad (30)$$

$$\vec{F}_1(\vec{\sigma}) = -\frac{e\hbar}{4mc^2}\dot{\vec{r}} \times (\vec{\nabla} \times (\vec{\sigma} \times \vec{\nabla}V(\vec{r})) + \frac{e\hbar}{4mc^2}\dot{\vec{r}} \times (\vec{\nabla} \times (\vec{\sigma} \times \vec{\nabla}V_a(r))) \quad (31)$$

$$\vec{F}_2(\vec{\sigma}, \theta) = -\frac{\theta e^2}{4mc^3}\dot{\vec{r}} \times (\vec{\nabla} \times (\vec{\sigma} \times \vec{\nabla}V(\vec{r})) + \frac{\theta e^2}{4mc^3}\dot{\vec{r}} \times (\vec{\nabla} \times (\vec{\sigma} \times \vec{\nabla}V_a(r))) \quad (32)$$

Here we have neglected $1/c^4$ terms. We can easily compare the form of the force obtained in (21), with the Lorentz force on the charge e . The spin dependent part of this effective Lorentz force, *i.e.* $\vec{F}_1(\vec{\sigma})$ and $\vec{F}_2(\vec{\sigma}, \theta)$ are responsible for the inertial spin current of the system on the NC space. One can write the total Lorentz force as

$$\vec{F}_0 + \vec{F}_{\vec{\sigma}} = e\vec{E}_{tot} + \frac{e}{c}(\dot{\vec{r}} \times \vec{B}(\vec{\sigma})), \quad (33)$$

where the spin Lorentz force is

$$\vec{F}_{\vec{\sigma}} = \frac{e}{c}(\dot{\vec{r}} \times \vec{B}(\vec{\sigma})). \quad (34)$$

This spin dependent Lorentz force is responsible for the spin current produced in the system. $\vec{B}(\vec{\sigma})$, the *effective magnetic field* appearing in the spin space can be read as

$$\vec{B}(\vec{\sigma}) = \vec{\nabla} \times \vec{A}(\vec{\sigma}) \quad (35)$$

$$= \vec{\nabla} \times (\vec{A}_1(\vec{\sigma}) + \vec{A}_2(\vec{\sigma}, \theta)), \quad (36)$$

where the forms of the vector potentials are explicitly given by

$$\vec{A}_1(\vec{\sigma}) = -\frac{\hbar}{4mc}(\vec{\sigma} \times \vec{\nabla}V(\vec{r})) + \frac{\hbar}{4mc}(\vec{\sigma} \times \vec{\nabla}V_a(r)) \quad (37)$$

$$\vec{A}_2(\vec{\sigma}, \theta) = -\frac{\theta e}{4mc^2}(\vec{\sigma} \times \vec{\nabla}V(\vec{r})) + \frac{e\theta}{4mc^2}(\vec{\sigma} \times \vec{\nabla}V_a(r)). \quad (38)$$

Finally, the Hamiltonian (14) can be written as

$$H_{FW*} = \frac{1}{2m}(\vec{p} - \frac{e}{c}\vec{A}(\vec{\sigma}))^2 + eV_{tot} \quad (39)$$

Proceeding further to deduce the spin current, the averaging methodology followed in [31] is adopted. From eqn. (21), it is observed that the contribution from $\vec{F}(\vec{\sigma})$ in comparison to \vec{F}_0 is very small. One can treat this as a perturbation in the next part of our calculation [31]. Breaking into different parts, the solution of equation (20) can be written as $\dot{\vec{r}} = \dot{\vec{r}}_0 + \dot{\vec{r}}_{\vec{\sigma}}$, where $\dot{\vec{r}}_{\vec{\sigma}}$ has two parts, arising from the effect of force $\vec{F}_1(\vec{\sigma})$, $\vec{F}_2(\vec{\sigma}, \theta)$. So we can write

$$\dot{\vec{r}}_{\vec{\sigma}} = \dot{\vec{r}}_1(\vec{\sigma}) + \dot{\vec{r}}_2(\vec{\sigma}, \theta) \quad (40)$$

If the relaxation time τ is independent of $\vec{\sigma}$ and for the constant total electric field \vec{E}_{eff} , following [31] we can write,

$$\langle \dot{\vec{r}}_0 \rangle = -\frac{\tau}{m} \left\langle \frac{\partial V_{eff}}{\partial r} \right\rangle = \frac{e\tau}{m} \vec{E}_{eff}, \quad (41)$$

where we denote $\vec{E}_{eff} = -e\vec{\nabla} (V_0(\vec{r}) - V_a(\vec{r}))$.

$$\langle \dot{\vec{r}}_1(\vec{\sigma}) \rangle = -\frac{\hbar e^2 \tau^2}{2m^3 c^2} \vec{E}_{eff} \times \left\langle \frac{\partial}{\partial r} \times (\vec{\sigma} \times \frac{\partial V_l}{\partial r}) \right\rangle + \frac{\hbar \tau^2 e^2}{2m^3 c^2} \vec{E}_{eff} \times \left\langle \frac{\partial}{\partial r} \times (\vec{\sigma} \times \frac{\partial V_a}{\partial r}) \right\rangle \quad (42)$$

$$\langle \dot{\vec{r}}_2(\vec{\sigma}, \theta) \rangle = -\frac{\theta e^3 \tau^2}{4m^3 c^3} \vec{E}_{eff} \times \left\langle \frac{\partial}{\partial r} \times (\vec{\sigma} \times \frac{\partial V_l}{\partial r}) \right\rangle + \frac{e^3 \tau^2 \theta}{4m^3 c^3} \vec{E}_{eff} \times \left\langle \frac{\partial}{\partial r} \times (\vec{\sigma} \times \frac{\partial V_a}{\partial r}) \right\rangle \quad (43)$$

In the above expression of $\langle \dot{\vec{r}}_1(\vec{\sigma}) \rangle$ and $\langle \dot{\vec{r}}_2(\vec{\sigma}, \theta) \rangle$, a term is present which represents the volume average of electrostatic potential $\partial_i \partial_j V_l(r)$. In the study [31] of the spin Hall effect of *Al*, which is a cubic lattice, the only contribution permitted by symmetry is

$$\left\langle \frac{\partial^2 V_l}{\partial r_i \partial r_j} \right\rangle = \mu \delta_{ij}, \quad (44)$$

where μ is a constant depending on the system. However, on NCS, within the first order correction [26], we can use (44) and the value of $[\vec{A}, V(\vec{r})]_*$ to obtain,

$$\langle \dot{\vec{r}}_1(\vec{\sigma}) \rangle = \frac{\hbar e^2 \tau^2 \mu}{4m^3 c^2} (\vec{\sigma} \times \vec{E}_{eff}) \quad (45)$$

and

$$\langle \dot{\vec{r}}_2(\vec{\sigma}, \theta) \rangle = \frac{\theta e^3 \tau^2 \mu}{4m^3 c^3} (\vec{\sigma} \times \vec{E}_{eff}) \quad (46)$$

One may note that the second term in the right hand side of (42) and (43) vanishes for constant acceleration and so contribution from that term is zero in (45) and (46). It is interesting to notice that this term will contribute when the system is under non linear acceleration.

To calculate the spin current we now introduce the spin polarization vector $\vec{\lambda} = \langle \vec{\sigma} \rangle$. The density matrix of the charge carriers in spin space can be written as

$$\rho^s = \frac{1}{2} \rho (1 + \vec{\lambda} \cdot \vec{\sigma}) \quad (47)$$

where ρ is the total charge concentration. Using eqn (47) and the eqns (41), (45) and (46) we derive the total spin current as

$$\vec{j} = e \langle \rho^s \vec{r} \rangle = \vec{j}^{o,a} + \vec{j}^s(\vec{\sigma}) + \vec{j}^s(\vec{\sigma}, \theta) \quad (48)$$

The various components of this current are given by

$$\vec{j}^{o,a} = \frac{e^2 \tau \rho}{m} \vec{E}_{eff}, \quad (49)$$

$$\vec{j}^s(\vec{\sigma}) = \left(\frac{\hbar e^3 \tau^2 \rho \mu}{2m^3 c^2} \right) (\vec{\lambda} \times \vec{E}_{eff}), \quad (50)$$

$$\vec{j}^s(\vec{\sigma}, \theta) = \left(\frac{e^4 \tau^2 \mu \rho \theta}{2m^3 c^3} \right) (\vec{\lambda} \times \vec{E}_{eff}) \quad (51)$$

of which is $\vec{j}^s(\vec{\sigma})$ is θ independent and $\vec{j}^s(\vec{\sigma}, \theta)$ is θ dependent part of current.

The term $\vec{j}^{o,\vec{a}}$ has two parts as \vec{j}^o and $\vec{j}^{\vec{a}}$, where

$$\vec{j}^o = \frac{e^2 \tau \rho}{m} \vec{E} \quad (52)$$

$$\vec{j}^{\vec{a}} = \frac{e^2 \tau \rho}{m} \vec{E}_{\vec{a}}. \quad (53)$$

In the expression of (53), we know that $\vec{E}_{\vec{a}} = \frac{m\vec{a}}{e}$. So when there is no acceleration in the system $\vec{E}_{\vec{a}}$ becomes zero and there will be no spin current due to acceleration. Inserting the value of $\vec{E}_{\vec{a}}$ in (53) one can get

$$\vec{j}^{\vec{a}} = e\tau\rho\vec{a} = e\tau\rho a\hat{a}, \quad (54)$$

where $\vec{a} = a\hat{a}$, \hat{a} is the unit vector in the direction of $\vec{E}_{\vec{a}}$. Thus one can define a parameter which is analogue of the *conductivity* as

$$\sigma_H^{\vec{a}} = e\tau\rho a \quad (55)$$

which appears as an effect of the effective electric field generated due to the inertial effect of linear acceleration in the system. We can derive the corresponding spin Hall conductivities from the expressions of the spin currents (49), (50) and (51) respectively as

$$\begin{aligned} \sigma_H &= \frac{e^2 \tau \rho}{m} \\ \sigma_H^s &= \frac{\hbar e^3 \tau^2 \rho \mu}{2m^3 c^2} \\ \sigma_{H\theta}^s &= \frac{e^4 \tau^2 \mu \rho \theta}{2m^3 c^3} \end{aligned} \quad (56)$$

$\sigma_{H\theta}^s$ is the conductivity arising due to the NC correction. If we consider the effective electric field acts in the z direction, the component of spin current are as follows

$$\hat{j}_x^s(\vec{\sigma}) = (\sigma_H^s + \sigma_{H\theta}^s) E_{eff} \quad (57)$$

$$\hat{j}_y^s(\vec{\sigma}) = -(\sigma_H^s + \sigma_{H\theta}^s) E_{eff}, \quad (58)$$

where E_{eff} is the absolute value of \vec{E}_{eff} . It is worthwhile to point out here that our result differs from a similar result of [26] because in [26] the authors took an explicit form for \vec{A} that can be trivially gauged away.

The non commutative correction ratio R can be written as

$$R = \frac{\sigma_{H\theta}^s}{\sigma_H^s} = \frac{e\theta}{c\hbar} \quad (59)$$

Now from (56) we can evaluate the ratio of charge and spin hall conductivities as

$$\frac{\sigma_H^s + \sigma_{H\theta}^s}{\sigma_H} = \frac{\hbar e \tau \mu}{2m^2 c^2} \left(1 + \frac{e\theta}{c\hbar}\right) = \frac{\hbar e \tau \mu}{2m^2 c^2} (1 + R). \quad (60)$$

If we put θ equal to zero in (60), the result is similar as [31]. Our results can yield a bound for θ provided R - the ratio of conductivities are experimentally measurable.

III. SPIN POLARIZATION IN NC SPACE

In this section, we consider only the θ dependent terms, in the Dirac Hamiltonian (14) for a linearly accelerating frame.

$$H_\theta = \frac{\vec{p}^2}{2m} + \frac{ie^2}{4m^2 c^3} \vec{\sigma} \cdot ([\vec{A}, V_a(\vec{r})]_* \times \vec{p}) \quad (61)$$

In terms of the *induced effective field* $\vec{E}_{\vec{a},\theta}$ the spin-orbit Hamiltonian for the NC part only is obtained as

$$H_\theta = \frac{\hbar^2 \vec{k}^2}{2m} + \frac{e\hbar^2}{4m^2 c^2} \vec{\sigma} \cdot (\vec{k} \times \vec{E}_{\vec{a},\theta}) \quad (62)$$

where $\vec{E}_{\vec{a},\theta}$ represents the induced effective electric field present in the system due to acceleration and noncommutativity and $\vec{p} = \hbar \vec{k}$ is the crystal momentum, m is the electrons mass.

The spin-orbit coupling due to combined action of noncommutativity and linear acceleration, can now be analyzed by the same framework as the standard Hamiltonian with SOC. The special relativity arguments can qualitatively explain the effect of SOC. For electrons moving through a lattice, the electric field \vec{E} is Lorentz transformed to an effective magnetic field $(\vec{k} \times \vec{E}) \approx \vec{B}(\vec{k})$ in the rest frame of the electron. Thus from (62) we can write

$$H_\theta = \frac{\hbar^2 \vec{k}^2}{2m} + \gamma \vec{\sigma} \cdot \vec{B}_{\vec{a},\theta}(\vec{k}). \quad (63)$$

Here, γ is the coupling strength and $\vec{B}_{\vec{a},\theta}$ is the effective magnetic field in the momentum space due to $\vec{E}_{\vec{a},\theta}$. For each value of \vec{k} , the spin degeneracy of electrons split into two subbands $|\pm\rangle$.

In laboratory frame the magnetic field axis rotates as one moves from one point to other. A local transformation may rotate the frame in such a way that the spin axis (along z axis) points along the unit vector $\hat{n}_{a,\theta}(\vec{k})$, where

$$\hat{n}_{a,\theta}(\vec{k}) = \frac{\vec{B}_{\vec{a},\theta}(\vec{k})}{|\vec{B}_{\vec{a},\theta}(\vec{k})|}. \quad (64)$$

Here *hat* above n denotes that $\hat{n}_{a,\theta}$ is a unit vector.

For a particular choice $\vec{a} = (0, 0, a_z \hat{z})$, $\vec{E}_{\vec{a},\theta} = (0, 0, \frac{e\theta}{c\hbar} \vec{E}_{a,z} \hat{z})$, we can write $\vec{B}_{\vec{a},\theta}(\vec{k}) = (\frac{e\theta}{c\hbar})(a_z k_y, -a_z k_x)$. From (63), the SOC Hamiltonian for this accelerating frame in NC space can be written as

$$H_\theta = \frac{\hbar^2 \vec{k}^2}{2m} - \alpha \frac{e\theta}{c\hbar} (k_x \sigma_y - k_y \sigma_x) \quad (65)$$

$$= \frac{\hbar^2 \vec{k}^2}{2m} - \alpha_\theta (k_x \sigma_y - k_y \sigma_x) \quad (66)$$

where

$$\alpha_\theta = \alpha \frac{e\theta}{c\hbar} = R\alpha. \quad (67)$$

Here $R = \frac{e\theta}{c\hbar}$ and α contains the effect of a_z only [21], whereas α_θ is the coupling strength of the accelerating system on the NC space. The spin-orbit Hamiltonian (65) is similar to the well known Rashba Hamiltonian [32] where the coefficient α_θ represents a Rashba like coupling in NC space with the presence of acceleration. However, in the usual Rashba coupling, structural inversion asymmetry is responsible for the generation of internal field, whereas in our formulation, responsibility lies with the electric field induced due to acceleration further modified by NC effects. Due to the presence of noncommutativity the strength of the coupling constant α modified to α_θ in an accelerating system. Again as expected, it is interesting to note that for $\theta = 0$ in the above equations we retrieve back the same result as in [21]. As the Rashba coupling strength have enormous effects on condensed matter systems, we can view a lot of interesting results with this Rashba like coupling strength in NC framework.

Dealing of the spin-orbit Hamiltonian in a NC space with a time dependent acceleration $\vec{a}(t)$ provides us with some interesting results. For a time dependent acceleration [24] the induced effective time dependent electric field, given by $\vec{E}_{\vec{a}}$, is also time dependent. Thus the time dependent Hamiltonian (63) is

$$H_\theta(t) = \frac{\hbar^2 \vec{k}^2}{2m} + \gamma \vec{\sigma} \cdot \vec{B}_{\vec{a},\theta}(\vec{k}(t)). \quad (68)$$

Finally, the expression for out of plane spin polarization in the NC space can be derived. The acceleration of the carriers along with the time dependence of the spin orbit Hamiltonian generates an additional component [21, 34] $\vec{B}_\perp = (\vec{n}_{\vec{a},\theta} \times \hat{n}_{\vec{a},\theta})$, in addition to the effective magnetic field $\vec{B}_{\vec{a},\theta}(\vec{k})$, which basically explains the origin of the out

of plane spin polarization. Thus following the methodology in [34], the out of plane spin polarization in NC space is given by

$$s_z^\theta \approx \pm \frac{1}{|\vec{B}_{\vec{a},\theta}(\vec{k})|} \frac{\hbar}{2} (\vec{n}_{\vec{a},\theta} \times \hat{n}_{\vec{a},\theta}) \cdot \hat{z} \quad (69)$$

$$= \pm \frac{\hbar^2}{2\alpha_\theta p} \frac{\hbar}{2} (\vec{n}_{\vec{a},\theta} \times \hat{n}_{\vec{a},\theta}) \cdot \hat{z} \quad (70)$$

The above expression shows a very important result of our calculation. It states that if the electron is simultaneously in a accelerating frame and also in a non commutative space, the out of plane spin polarization depends on a θ dependent Rashba like coupling parameter α_θ . We get back the results of [21], when we substitute $\theta = 0$. One should notice here that the expression of s_z^θ depends on the exact configuration of the unit vector $\hat{n}_{\vec{a},\theta}$ and the time derivative of $\hat{n}_{\vec{a},\theta}$.

IV. DISCUSSION AND A NEW BOUND FOR θ

Using the expression of the out of plane spin polarization in this section we want to propose a new bound for θ . In the commutative sector the out of plane spin polarization vector is given by [21]

$$s_z \approx \pm \frac{\hbar^2}{2\alpha p} \frac{\hbar}{2} (\vec{n}_{\vec{a}} \times \hat{n}_{\vec{a}}) \cdot \hat{z} \quad (71)$$

As the θ parameter is considered as a constant, the direction of $\hat{n}_{a,\theta}$ in (64) is same as \hat{n}_a in the reference [21]. We can then write the NC correction ratio by comparing the expressions of (69) and (71) as

$$Q = \frac{s_z}{s_z^\theta} = \frac{\alpha_\theta}{\alpha} = \frac{e\theta}{c\hbar} = R. \quad (72)$$

A new bound for θ can be set by considering Q to be of the order of one. Thus we can set a lower limit for θ , using the relation (72) as $\frac{1}{\sqrt{\theta}} \geq 10^{-12} \text{Gev}$, which matches exactly with the bound mentioned in the reference [26]. This is one of our major results.

V. CONCLUSION

To conclude, a very general framework of a Dirac electron in external electromagnetic field has been considered that lives in a non-inertial as well as NC frame. The non-inertial effects are introduced in the Dirac equation and subsequently Foldy-Wouthuysen transformations are exploited to reduce the system to a non-relativistic regime. Subsequently non-commutativity effects are introduced by replacing commutator brackets by *-commutator brackets (or Moyal brackets) and the dynamics of the resulting system is analyzed in detail from a spin Hall effect perspective. In particular, generalized expressions for the spin current and subsequent spin Hall conductivities are computed to the lowest non-trivial order in θ - the non-commutativity parameter. The intriguing part of our result is the fact that the acceleration \vec{a} and non-commutative θ effects are explicitly entangled in the spin current expressions (50,51) however, surprisingly, this is not manifested in spin Hall conductivities (57) (as they are conventionally defined). The non-inertial and non-commutative effects in spin Hall conductivities are mutually exclusive, (at least to the order of approximation we have considered). Here the experimentally relevant part of our analysis is that we get a θ dependent Rashba like coupling parameter in the non commutative space which causes a decrement of the out of plane spin polarization. We have suggested a novel way of providing a bound for θ through s_z^θ that can be subjected to experiment. Our predicted numerical value for the θ -bound agrees with existing results.

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